SINGLE OPTION CORRECT

1. If the sum of first n terms of an A.P. is cn(n - 1), where $c \neq 0$, then sum of the squares of these terms is

(A) $c^2n^2(n+1)^2$ (B) $\frac{2}{3}c^2n(n-1)(2n-1)$ (C) $\frac{2}{3}c^2n(n+1)(2n+1)$ (D) $\frac{c^2n^2}{3}(n+1)^2$

2. If $S_n = 1+\frac{1}{2}+\frac{1}{2^2}+........+\frac{1}{2^{n-1}}$, then the least integral value of n such that $2-S_n < \frac{1}{100}$ is

(A) 7 (B) 9 (C) 8 (D) 6

3. If $T_n = (n^2+1)\cdot n!$ and $S_n = T_1 + T_2 + T_3 + ...+ T_n$. Let $\frac{T_{10}}{S_{10}} = \frac{a}{b}$ are relatively prime natural numbers, then the value of (b - a) is

(A) 8 (B) 9 (C) 10 (D) 11

4. If $\sum_{r=1}^{n} r^3 - \sum_{p=1}^{n}\sum_{m=1}^{p}\sum_{r=1}^{m} 1 = 80$, then possible value of n can be –

(A) 3 (B) 4 (C) 5 (D) 6

5. The value of $\sum_{k=1}^{\infty}\frac{3k^2+3k+1}{(k^2+k)^3}$ is equal to

(A) 1/8 (B) 1/4 (C) 1/2 (D) 1

6. If n arithmetic means $A_1, A_2, ..., A_n$ are inserted between 50 and 100 and n harmonic means $H_1, H_2, H_3, ..., H_n$ are inserted between the same two numbers, then A_2H_{n-1} is equal to

(A) 5000 (B) 10000/n (C) 10000 (D) 250/n

7. Let $a_0 = \frac{5}{2}$ and $a_k = a^2_{k-1} - 2$ for $k \geq 1$, then the value of $\prod_{k=0}^{\infty}\left(1-\frac{1}{a_k}\right)$ is-

(A) 1/5 (B) 2/5 (C) 3/7 (D) 4/7

8. The value of $\sum_{n=2}^{\infty}\frac{n}{1+n^2(n^2-2)}$ is equal to

(A) 5/4 (B) 1 (C) 5/16 (D) ¼

9. If p times the pth term of an A.P. is equal to q times the qth term of an A.P., then (p + q)th term is?

(A) 0 (B) 1 (C) 2 (D) 3

10. The geometric series $a + ar + ar^2 + ar^3 +........\infty$ has sum 7 and the terms involving odd powers of r has sum '3', then the value of $(a^2 - r^2)$ is –

(A) 5/4 (B) 5/2 (C) 25/4 (D) 5

11. Consider a sequence whose sum of first n-terms is given by $S_n = 4n^2 + 6n, n \in N$, then T_{15} of this sequence is –

(A) 118 (B) 120 (C) 122 (D) 86

12. Let a_n be a sequence such that $a_1 = 5$ and $a_{n+1} = a_n + (n - 2)$ for all $n \in N$, then a_{51} is

(A) 1165 (B) 1170 (C) 1175 (D) 1180

13. If b > 0, then minimum value of $\frac{1+b^2+b^3+b^4+8b^5+b^6+b^7+b^{13}}{b^5}$ is equal to-

(A) $8^{9/8}$ (B) 7 (C) 64 (D) 15

14. If G_1 & G_2 are two geometric means &A is A.M inserted between two numbers, then value of $\frac{G_1^2}{G_2}+\frac{G_2^2}{G_1}$ is

(A) $\frac{A}{2}$ (B) A (C) 2A (D) 3A

15. If $S=1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+.....\infty$, then $s=1+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}+.....\infty$ is equal to:

(A) $\frac{S}{2}$ (B) $\frac{3S}{4}$ (C) $S-\frac{1}{4}$ (D) $S-\frac{1}{2}$

16. Let $a_1, a_2, a_3, a_4, ..., a_{11}$ be a geometric sequence. If $\prod_{k=1}^{11} a_k = 6$, then the value of $(a_5 a_6 a_7)$ is equal to:

(A) $6^{5/11}$ (B) $25^{1/11}$ (C) $216^{1/11}$ (D) $343^{1/11}$

17. The sum of the series, $1+2\cdot\left(1+\frac{1}{n}\right)+3\cdot\left(1+\frac{1}{n}\right)^2+4\cdot\left(1+\frac{1}{n}\right)^3+......\infty$ where $\left|1+\frac{1}{n}\right|<1$, is:

(A) n^2 (B) n(n + 1) (C) $n\left(1+\frac{1}{n}\right)^2$ (D) (n + 1)(n + 2)

18. Let $F_0 = 0, F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ $(n \geq 2)$, then the value of the sum $\sum_{n=1}^{\infty}\frac{F_n}{3^n}$ is:

(A) 3/5 (B) 1/3 (C) 2/3 (D) 7/9

19. The co-efficient of x^{15} in the product $(1 - x)(1 - 2x)(1 - 2^2 x)(1 - 2^3 x).......(1 - 2^{15}x)$ is equal to

(A) $2^{105} - 2^{121}$ (B) $2^{121} - 2^{105}$ (C) $2^{120} - 2^{104}$ (D) $2^{104} - 2^{120}$

20. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is

(A) $\alpha - \beta$ (B) $\frac{\alpha-\beta}{100}$ (C) $\beta - \alpha$ (D) $\frac{\alpha-\beta}{200}$

21. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If a < b < c and a + b + c = 3/2, then value of a is

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

22. Let $S_k = \sum_{i=0}^{\infty}\frac{1}{(k+1)^i}$, then $\sum_{k=1}^{n} k\cdot S_k$ equal:

(A) $\frac{n(n+1)}{2}$ (B) $\frac{n(n-1)}{2}$ (C) $\frac{n(n+2)}{2}$ (D) $\frac{n(n+3)}{2}$

23. If x, y, z > 0 and x + y + z = 1 then $\frac{xyz}{(1-x)(1-y)(1-z)}$ is necessarily

(A) ≥ 8 (B) $\leq 1/8$ (C) 1 (D) None of these

24. The sum of the series $1+3x+6x^2+10x^3+.......\infty$ will be (where $|x|<1$)

(A) $\frac{1}{(1-x)^2}$ (B) $\frac{1}{1-x}$ (C) $\frac{1}{(1+x)^2}$ (D) $\frac{1}{(1-x)^3}$

25. The number of common terms to the two sequences 17, 21, 25,……., 417 & 16, 21, 26,……, 466 is

(A) 21 (B) 19 (C) 20 (D) 91

26. It's given that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+.......\infty=\frac{\pi^4}{90}$. Then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+.......\infty$ is equal to

(A) $\frac{\pi^4}{96}$ (B) $\frac{\pi^4}{45}$ (C) $\frac{89\pi^4}{90}$ (D) none of these

27. If $S_n=\sum_{r=1}^{n}\left(\frac{2r+1}{r^4+2r^3+r^2}\right)$ then S_{20} is equal to

(A) $\frac{220}{221}$ (B) $\frac{420}{441}$ (C) $\frac{439}{221}$ (D) $\frac{440}{441}$

28. If $1,\frac{x}{2},y$ are in harmonic progression $(x,y\neq 0)$, then the number of integral ordered pair (x,y) is

(A) 8 (B) 3 (C) 4 (D) 5

29. If $\sum_{r=1}^{n}\frac{r}{1.3.5.7......(2r+1)}=\frac{a}{b}\left(1-\frac{c}{1.3.5......(2n+1)}\right)$ Where $a,b,c\in N$, $a<b$ & b is a prime number then the value of (a+ b + c) is

(A) 1 (B) 2 (C) 3 (D) 4

30. Let a, b, c are in AP & $|a|,|b|,|c|<1$ if $x=1+a+a^2+...\text{to }\infty$, $y=1+b+b^2+.........\infty$, $z=1+c+c^2+.........\infty$, then x, y, z are in

(A) AP (B) GP (C) HP (D) None

31. If a, b, c, d, e be 5 numbers such that a, b, c are in AP, b, c, d are in GP, c, d, e are in HP. Then a, c, e are in

(A) AP (B) GP (C) HP (D) None

32. The first term of an infinitely decreasing GP is unity and its sum is S. The sum of the squares of the terms of the progression is

(A) $\frac{S}{2S-1}$ (B) $\frac{S^2}{2S-1}$ (C) $\frac{S}{2-S}$ (D) S^2

33. Consider an AP $a_1,a_2,a_3.....$ such that $a_3+a_5+a_8=11$ and $a_4+a_2=-2$, then the value of $a_1+a_6+a_7$ is

(A) - 8 (B) 5 (C) 7 (D) 9

34. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation $2A+G^2=27$, then the numbers are

(A) 3 & 6 (B) 2 and 9 (C) 9 and 4 (D) 6 and 4

35. Let $b_1, b_2, \ldots\ldots, b_n$ be a geometric sequence such that $b_1 + b_2 = 1$ and $\sum_{k=1}^{\infty} b_k = 2$. Given that $b_2 < 0$, then the value of b_1 is

(A) $2 - \sqrt{2}$ (B) $1+\sqrt{2}$ (C) $2 + \sqrt{2}$ (D) $4 + \sqrt{2}$

36. Let 1, 4, 7, ……. and 9, 16, 23, ….. be two arithmetic progressions. The set S is the union of the first 2019 terms of each sequence. How many distinct numbers are in S?

(A) 3650 (B) 3750 (C) 3800 (D) 3850

37. Given the progression $10^{1/11}, 10^{2/11}, 10^{3/11}, 10^{4/11}, \ldots\ldots, 10^{n/11}$. The least positive integer n such that the product of the first n terms of the progression exceeds 100,000 is

(A) 8 (B) 9 (C) 10 (D) 11

38. Suppose that $\{a_n\}$ is an arithmetic sequence with $a_1 + a_2 + a_3 + \ldots\ldots + a_{100} = 100$ and $a_{101}+a_{102}+\ldots + a_{200} = 200$. What is the value of $a_2 - a_1$?

(A) 0.0001 (B) 0.001 (C) 0.01 (D) 1

39. The first four terms in an arithmetic sequence are $x + y$, $x - y$, xy and $\frac{x}{y}$ in the order. What is fifth term?

(A) $-\frac{15}{8}$ (B) $-\frac{6}{5}$ (C) 0 (D) $\frac{123}{40}$

40. 150 worker were engaged to finish a piece of work in a certain number of days. Four workers stopped working on the second day, four more workers stopped working on the third day and so on. It took 8 more days to finish the work, then the number of days in which the work was completed is

(A) 29 days (B) 24 days (C) 25 days (D) none of these

41. Let $E = x^{2017} + y^{2017} + z^{2017} - 2017\,xyz$ (where $x, y, z \geq 0$) then the least value of E is –

(A) 0 (B) - 2014 (C) - 2017 (D) 2017

42. If $a_1, a_2, \ldots\ldots a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \ldots a_{n-1} + 2\,a_n$ is –

(A) $n(2c)^{1/n}$ (B) $(n+1)c^{1/n}$ (C) $2nc^{1/n}$ (D) $(n+1)(2c)^{1/n}$

43. The number $\underbrace{111\ldots\ldots1}_{91\ times}$ is a

(A) Prime number (B) multiple of 7 (C) Divisible by $\frac{10^7-1}{9}$ (D) Divisible by $\frac{10^5-1}{9}$

44. Sum of the series: $\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4} + \ldots\ldots$ up to ∞ is

(A) 65/36 (B) 36/65 (C) 55/36 (D) none of these

MULTIPLE OPTIONS CORRECT

1. Let $x_1, x_2, \ldots\ldots, x_{4001}$ are in harmonic progression and $x_1x_2 + x_2x_3 + \ldots\ldots + x_{4000}x_{4001} = 10$ and $\frac{1}{x_2} + \frac{1}{x_{4000}} = 50$, then-

(A) $\sum_{r=1}^{4001} \frac{1}{x_r} = 100025$ (B) $\sum_{r=1}^{4001} \frac{1}{x_r} = 100000$ (C) $\left|\frac{1}{x_{4001}} - \frac{1}{x_1}\right| = 30$ (D) $\left|\frac{1}{x_{4001}} - \frac{1}{x_1}\right| = 40$

2. Let $S_n = \sum_{r=2}^{n} \frac{3^{r-1}(2r-3)}{r(r-1)}$, then

(A) S_9 is divisible by 4
(B) S_9 is divisible by 21
(C) $10\,S_{10} + 3 = 3^{10}$
(D) $7(S_7 + 3)$, $10(S_{10} + 3)$, $13(S_{13} + 3)$ are in G.P.

3. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s):

(A) 1056
(B) 1088
(C) 1120
(D) 1332

4. If 9th, 13th and 15th terms of an arithmetical progression are the first three terms of a geometric series whose sum of infinite terms is 80, then which of the following hold(s) good?

(A) Sum of the first 16 terms of the geometric progression is 860.
(B) First term of the arithmetic progression is 80.
(C) First term of the geometric progression is 40.
(D) If d is the common difference of arithmetic progression and r is the common ratio of geometric progression then $dr = -\frac{5}{2}$.

5. Let a_1, a_2, a_3, a_4 & a_5 be such that a_1, a_2 & a_3 are in AP, a_2, a_3 & a_4 are in GP & a_3, a_4 & a_5 are in HP, then a_1, a_3 & a_5 are not in

(A) GP
(B) AP
(C) HP
(D) AGP

6. If a, b, c, d are four distinct positive real numbers in harmonic progression. Then

(A) a + d > b + d
(B) ad > bc
(C) $a + \frac{1}{a} \geq 2$
(D) If a+b+c = 6, then $a^2bc^3\big|_{max} = 108$

7. If $T_1, T_2, T_3, \ldots\ldots\ldots\ldots T_{100}$ are in AP, then

(A) $\frac{T_{15} + T_{27}}{2} = T_{21}$
(B) $T_{15} + T_{28} = T_{18} + T_{25}$
(C) $T_1 + T_2 + .. + T_{2k} = k(T_3 + T_{2k-2}), k < 50$
(D) 63th term from last = T_{38}

8. Consider $S_\infty = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots\ldots\infty$, then

(A) $S_\infty = \frac{x}{x-1}\ \forall\, |x| < 1$
(B) $S_\infty = \text{Not Define, if } x = 1$
(C) $S_\infty = \frac{3}{2}$, if x = 3
(D) $S_\infty > 0\ \forall\ x \in (-\infty, -1) \cup (1, \infty)$

9. If $2x,\ x+8,\ 3x+1$ are first three term of an A.P. then which of the following statement is/are correct

(A) x = 5
(B) $T_7 = 28$
(C) $S_{10} = 235$
(D) common diff. = 3

10. Ratio of sum of n terms of two distinct AP is $\frac{7n+1}{3n-1}$ then

(A) Ratio of their 6th term = 39/16
(B) Ratio of their 1st term = 4
(C) Ratio of their 5th term = 2
(D) None of these

11. If three positive unequal numbers a, b, c are in HP then

(A) $a + c > 2b$ (B) $a^2 + c^2 > 2b^2$ (C) $a^2 + c^2 > 2ac$ (D) $a^2 + c^2 < b^2$

12. For the AP given by $a_1, a_2, a_3, \ldots\ldots a_n, a_{n+1}, \ldots\ldots$ the equations satisfied are

(A) $a_1 + 2a_2 + a_3 = 0$ (B) $a_1 - 2a_2 + a_3 = 0$

(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$ (D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

13. Consider an infinite geometric series first term 'a' and common ratio 'r'. If the sum is 4 and the second term is $\frac{3}{4}$, then

(A) $a = 3$ (B) $a = \frac{3}{2}$ (C) $r = \frac{1}{4}$ (D) $r = \frac{1}{2}$

14. If a, b, c are in HP, then

(A) $\frac{a}{c} = \frac{a-b}{b-c}$ (B) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP

(C) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in AP (D) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

15. The sum of the first three consecutive terms of an AP 9, and the sum of their squares is 35. Then the sum to n terms of the series is

(A) $n(n+1)$ (B) n^2 (C) $n(4-n)$ (D) $n(6-n)$

16. The consecutive digits of a three digit number are in G.P. If the middle digit be increased by 2, then they form an A.P. If 792 is subtracted from this, then we get the number constituting of same three digits but in reverse order. Then number is divisible by

(A) 7 (B) 49 (C) 19 (D) 15

17. $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{m^2 n}{\left(n.3^m + m.3^n\right)} = \frac{p}{q}$ (where p, q are co-prime) then:

(A) $p + q = 41$ (B) $p < q$ (C) p is a perfect square (D) q is a perfect square

18. Given four positive numbers in A.P. If 5, 6, 9 and 15 are added respectively to these numbers, we get a G.P. then which of the following holds?

(A) The common ratio of G.P. is 3/2 (B) Common ratio of G.P. is 2/3

(C) First term of G.P. is 8 (D) Common difference of the A.P. is 3

INTEGER TYPE

1. If the sum of first n terms of an AP (having positive terms) is given by $S_n = (1 + 2T_n)(1 - T_n)$ where T_n is the n^{th} term of series then $T_2^2 = \frac{\sqrt{a} - \sqrt{b}}{4}$ (a, b ϵ N) Find the value of (a + b)

2. Suppose that all terms of an A.P. are natural numbers. If the ratio of the sum of first seven terms to the sum of first eleven terms is 6:11 and the seventh term lies in between 130 and 140, then the common difference of A.P. is:

3. Let $a_1, a_2, a_3, \ldots\ldots., a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4,……11. If $\frac{a_1^2 + a_2^2 + a_3^2 + \ldots\ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + a_3 + \ldots\ldots + a_{11}}{11}$ is equal to ____

4. If a, b, c are in AP then value of $\frac{(a-c)^2}{(b^2 - ac)}$ will be $(a \neq b \neq c)$

5. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11,…… and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23,….. Then, the number of elements in the set X ∪ Y is ___

6. The sum of first three consecutive terms of a decreasing GP is 19 & their product is 216. If the sum of infinite number of terms of the GP is K then $\frac{k}{3}$ is equal to ______

7. If a, b and c are three distinct positive real numbers, then the minimum integral value of the expression $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$ is equal to ______

8. If $3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \ldots\ldots \text{to } \infty = 8$, then the value of d is equal to _______

9. Let $A_1, A_2, A_3, \ldots, A_{51}$ are fifty one arithmetic means inserted between the numbers a and b. If value of $\frac{b + A_{51}}{b - A_{51}} - \frac{A_1 + a}{A_1 - a}$ is M, then remainder when M is divided by 10 is

10. Find the sum of infinity decreasing GP, the sum of whose first three terms is equal to 7 and the product of those terms is 8.

11. Let $a_1, a_2, a_3 \ldots.. a_{10}$ be in AP & $h_1, h_2, h_3 \ldots\ldots. h_{10}$ be in HP. If $a_1 = h_1 = 2$, $a_{10} = h_{10} = 3$, then find the value of $a_4 h_7$.

12. Let $a_1, a_2, a_3, \ldots\ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let m = 5n. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is:

13. Let $<a_n>$ be an arithmetic sequence such that $\sum_{i=1}^{50} a_{2i-1} = 50$, then $\left| \sum_{j=1}^{50} (-1)^{\frac{j^2+j}{2}} a_{2j-1} \right|$ is equal to

SUBJECTIVE PROBLEMS

1. Let $\{a_n\}$ be a sequence such that $a_1 = 4$ and sum of first n terms is S_n and $S_{n+1} - 3S_n - 2n - 4 = 0 \; \forall \; n \in N$, Find a_n.

2. For an Integer n ≥ 3, Let S_n denotes the sum of the products of the integers from 1 to n taken three at a time. (For example $S_3 = 1 \times 2 \times 3 = 6$, $S_4 = 1 \times 2 \times 3 + 1 \times 2 \times 4 + 1 \times 3 \times 4 + 2 \times 3 \times 4 = 50$ and so on.) Then the value of of S_{10} = __________. Hence prove by Induction that $S_n = \frac{1}{48} n^2 (n+1)^2 (n^2 - 3n + 2)$.

3. Find the sum of the following series

(i) $1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\ldots$upto n terms

(ii) 1 + 3 + 7 + 15 + 31 + ……….upto n terms

(iii) 1 + 5 + 11 + 19 + 29 +……….. upto n terms

(iv) $1.2 + 2.3x + 3.4x^2 + 4.5x^3 + \ldots\ldots. \infty \; (|x| < 1)$

(v) 1.n + 2.(n - 1) + 3.(n - 2)+ ………+ n.1

(vi) 5 + 7 + 11 + 17 + 25 + ….. upto n terms

4. If x, y, z are positive real numbers, such that x + y + z = a, then prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq \frac{9}{a}$.

5. Prove the following inequalities

(i) $(a + b + c)(ab + bc + ca) > 9abc$, where a > 0, b > 0, c > 0.

(ii) If a + b + c = 1 then $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\geq\frac{3}{2}$, where a > 0, b > 0, c > 0.

(iii) $\frac{a_1}{a_2}+\frac{a_2}{a_3}+\frac{a_3}{a_4}+.......+\frac{a_{n-1}}{a_n}+\frac{a_n}{a_1}>n$, where $a_i > 0$ for i = 1, 2, 3,….., n

(iv) if $a_1a_2a_3....a_n = 1$ then $(1+a_1)(1+a_2)(1+a_3)...(1+a_n) \geq 2^n$, where $a_i > 0$ for i = 1, 2, 3,……, n

(v) $2^n > 1+n\sqrt{2^{n-1}}, n > 1$ & $n \in N$.

(vi) $\frac{ab}{c^3}+\frac{bc}{a^3}+\frac{ca}{b^3}>\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$, where a > 0, b > 0, c > 0.

(vii) If a > 0, b > 0, c > 0 find the Minimum value of $(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

(viii) $\frac{a^8+b^8+c^8}{a^3b^3c^3}\geq\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ where a > 0, b > 0, c > 0

(ix) if a, b, c represents sides of a triangle then prove that: $ab+bc+ca<a^2+b^2+c^2<2(ab+bc+ca)$.

(x) If 0 < a, b, c < 1 and a + b + c = 2 then prove that $\left(\frac{a}{1-a}\right)\times\left(\frac{b}{1-b}\right)\times\left(\frac{c}{1-c}\right)\geq 8$

(xi) For positive numbers a, b, c such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$. Find minimum value of (a - 1)(b - 1) (c - 1) Ans:8

6. Find the greatest value of x^3y^4 if 2x + 3y = 7 and x ≥ 0, y ≥ 0.

7. If a + b + c = 18, find the maximum value of $a^2b^3c^4$ where a > 0, b > 0, c > 0.

8. If $x_1, x_2, x_3, ……., x_n$ are in H.P. then prove that $x_1x_2 + x_2x_3 + x_3x_4 + …….+ x_{n-1}x_n = (n - 1)x_1x_n$

9. Calculate the following

(i) $\sum_{r=1}^{n}\left(x^r+\frac{1}{x^r}\right)^2$ (ii) $\prod_{r=1}^{\infty}6^{\left(\frac{1}{2^r}\right)}$ (iii) $\sum_{r=1}^{n}\left(\frac{r}{1+r^2+r^4}\right)$ (iv) $\sum\sum_{0\leq i<j\leq n} ij$

10. If $\sum_{k=1}^{n}(k^2+3k+3)(k+1)!=(2007)(2007)!-4$ then the value of n must be

11. If nine A.M.'s & nine H.M's are inserted between 2 & 3. Then prove that $A_i+\frac{6}{H_i}=5$ where i = 1, 2, …..9 & A_i, H_i are ith AM & HM respectively.

12. Find the sum of the following series

(i) $\sum_{r=1}^{n}\frac{r}{(r+1)!}$ (ii) $\sum_{k=1}^{360}\left(\frac{1}{k\sqrt{k+1}+(k+1)\sqrt{k}}\right)$

(iii) $\sum_{k=1}^{100}(k^2+1)\cdot k!$ (iv) $\sum_{n=1}^{9999}\frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})}$

(v) $\sum_{r=0}^{9}\frac{r^2}{r^2+(9-r)^2}$ (vi) $\sum_{r=0}^{n-1}\sin(rx)\cos((n-r)x)$

(vii) $\frac{3}{1\cdot 2}\times\frac{1}{2}+\frac{4}{2\cdot 3}\times\left(\frac{1}{2}\right)^2+\frac{5}{3\cdot 4}\times\left(\frac{1}{2}\right)^3+......+\text{ to n terms}$

(viii) $\sum_{n=1}^{\infty} n^2 e^{-n}$

(ix) $\sum_{n=0}^{1947}\frac{1}{2^n+\sqrt{2^{1947}}}$

(x) $\sum_{n=2}^{\infty}\sum_{r=2}^{\infty}\frac{1}{r^n\, r!}$

(xi) $\sum_{r=0}^{\infty}\frac{2^r}{a^{2^r}+1}$ where a > 1

13. The first term of an A.P. is 5, the last is 45, and the sum 400. Find the number of terms and common difference.
14. If $S_1, S_2, S_3, S_4, \ldots\ldots, S_p$ are the sums of n terms of 'p' arithmetic series whose first terms are 1, 2, 3, 4,….. and whose common difference are 1, 3, 5, 7,……; prove that $S_1 + S_2 + S_3 + S_4 + \ldots\ldots + S_p = \frac{np}{2}(np+1)$.
15. Let $f(n)=1\times 3\times 5\times 7\times....\times(2n-1)$. Find the remainder when f(1) + f(2) + f(3) + …..+f(2016) is divided by 100.
16. If $\sum_{r=1}^{n} I(r)=n\left(2n^2+9n+13\right)$, then find the sum $\sum_{r=1}^{n}\sqrt{I(r)}$.
17. Prove that $0.4\overline{23}=\frac{419}{990}$ using infinite series.
18. Find the sum $S=(x+y)+\left(x^2+xy+y^3\right)+\left(x^3+x^2y+xy^2+y^3\right)+....+\text{ to n terms}$.
19. If n A.M's are inserted between 20 & 80 such that first mean : last mean = 1 : 3 find the common difference of the corresponding AP.

MATRIX MATCH

1. Match the following

	COULUMN – I		COULUMN - II
A	If a, b, c, d, e are in HP then $\frac{ab+bc+cd+de}{ae}$ is equal to	P	1
B	Let T_r be the r^{th} term of an HP & If $T_2=3$ & $T_3=2$ then T_6 is equal to	Q	2
C	Let $a_1, a_2, a_3, \ldots\ldots, a_{10}$ be in AP & $h_1, h_2, h_3, \ldots.. h_{10}$ be in HP & if $a_1 = h_1 = 1$ & $a_{10} = h_{10} = 2$, then a_4h_7 is equal to	R	3
D	Let $H_1, H_2, H_3, \ldots\ldots\ldots, H_{12}$ are twelve harmonic mean between 3 & 6, then $\frac{1}{H_1}+\frac{1}{H_2}+\frac{1}{H_3}+\ldots\ldots\ldots+\frac{1}{H_{12}}$ is equal to	S	4

2. Match the following

	COULUMN – I		COULUMN - II
A	If $2^1 \cdot 2^{1/2} \cdot 2^{1/4} \cdot 2^{1/8} \cdots \infty > 2^x$ then greatest integral value of x is	P	8
B	If a, b, c $\in$ {0, 1, 2, 3} such that $a \neq b \neq c$ then maximum value of $\lvert a-b\rvert + \lvert b-c\rvert + \lvert c-a\rvert$ is	Q	3
C	If $a^2 + 4b^2 + c^2 - 2a - 4b - 4c + 6 = 0$ then the value of abc is, (where a, b, $c \in R$)	R	1
D	If $x^2 - 2x - k = 0$ possess integral roots then k may be	S	6

PARAGRAPH

The sequence $\{a_n\}$ is defined by the formula $a_0 = 4$ and $a_{n+1} = a_n^2 - 2a_n + 2$ for $n \geq 0$. Let the sequence $\{b_n\}$ is defined by the formula $b_0 = 1/2$ and $b_n = \frac{2a_0a_1a_2.......a_{n-1}}{a_n} \; \forall n \geq 1$

1. The value of a_{10} is

(A) $1 + 2^{1024}$ (B) $1 + 3^{1024}$ (C) 4^{1024} (D) 6^{1024}

2. The value of n for which $b_n = \frac{3280}{3281}$ is

(A) 2 (B) 3 (C) 4 (D) 6

ANSWER KEY & SOLUTION

SINGLE OPTION CORRECT

1. B	2. C	3. B	4. B
5. D	6. A	7. C	8. C
9. A	10. B	11. C	12. D
13. A	14. C	15. B	16. C
17. A	18. A	19. A	20. B
21. D	22. D	23. B	24. D
25. C	26. A	27. D	28. D
29. D	30. C	31. B	32. B
33. C	34. A	35. C	36. B
37. D	38. C	39. D	40. C
41. B	42. A	43. C	44. A

MULTI OPTIONS CORRECT

1. A, C	2. A, B, D	3. A, D	4. B, C, D
5. B, C, D	6. A, B, C	7. A, B, C, D	8. C, D
9. A, B, C, D	10. A, B	11. A, B, C	12. B, D
13. A, C	14. A, B, D	15. B, D	16. A, B, C
17. A, B, C	18. A, C, D		

INTEGER TYPE

1. 6	2. 9	3. 0	4.
5. 3748	6. 9	7. 7	8. 9
9. 2	10. 8	11. 6	12. 9 or 3
13. 2			

SUBJECTIVE

1. $a_n = 5\cdot 3^{n-1} - 1 \ \forall\, n \in N$.   2. 18150

3. (i)   (ii)   (iii)   (iv)
   (v)   (vi)

6.   7.

9. (i) $\left(\dfrac{x^{2n}-1}{x^2-1}\right)\left(x^2+\dfrac{1}{x^{2n}}\right)+2n$ (ii) 6

   (iii) $\dfrac{n^2+n}{2(n^2+n+1)}$ (iv) $\left(\dfrac{\sum_{r=1}^{n} r^3 - \sum_{r=1}^{n} r^2}{2}\right)$

10. 2005

12. (i)   (ii) 18/19   (iii) $100 \times 101!$ (iv) 9

    (v) 5   (vi) $\left(\dfrac{n-1}{2}\right)\sin(nx)$ (vii)   (viii) $\dfrac{e(e+1)}{(e-1)^3}$

    (ix)   (x) 3 – e   (xi) $\dfrac{1}{a-1}$

13. n = 16, d = 8/3   15.   16. $\sqrt{\dfrac{3}{2}}\left(n^2+3n\right)$

18. $\dfrac{1}{x-y}\left(\dfrac{x^2(1-x^n)}{1-x}-\dfrac{y^2(1-y^n)}{1-y}\right)$ 20.

MATRIX MATCH

1. A→ S, B → P, C → Q, D → R.   2. A → R, B → S, C → R, D → P, Q

PARAGRAPH

1. B   2. B